Supervisor review for the dissertation

THE CAUCHY PROBLEM FOR DIFFERENCE EQUATIONS IN LATTICE CONES AND GENERATING FUNCTIONS FOR ITS SOLUTIONS

by Sreelatha Chandragiri

The calculus of finite differences arose and developed as part of mathematical analysis in the work of such mathematicians as G. Leibniz, B. Taylor, D. Stirling and acquired the character of an independent mathematical discipline thanks to L. Euler (1755). The main problems of the theory of finite differences include interpolation of functions and summation of functions with a discrete change in the argument. The problem of summation is closely related to the theory of difference equations, which in the case of linear equations is well developed and has a completed form. Difference equations (recurrence relations) in combination with the methods of the theory of generating functions represent a powerful research apparatus in various fields of mathematics and, in particular, in enumerative combinatorial analysis.

The difficulties of the transition from the one-dimensional theory of difference equations to the multidimensional one are quite comparable to those that arise during the transition from ordinary differential equations to partial differential equations. For the space of solutions of the multidimensional difference equation, additional conditions ("initial", "boundary") are set that allow one to distinguish the unique from an infinite number of solutions. The general name of the additional conditions is "Cauchy data", and the resulting problem is the Cauchy problem for the multidimensional difference equation. In the paper of M. Bousquet-Melou, M. Petkovsek (2000), the term Cauchy problem was not used, but various options for setting additional conditions for a multidimensional difference equation with constant coefficients in the positive octant of an integer lattice were studied, the existence and uniqueness of the solution was proved, and application examples were found the results obtained in an enumerative combinatorial analysis.

In the papers of E.K. Leinartas, M.S. Rogozina (Apanovich), A.P. Lyapin, T.I. Nekrasova (Yakovleva) (2004-2016) considered the problem of correctly posing the Cauchy problem for a multidimensional difference equation in unimodular rational cones of an integer lattice, we obtained formulas in which the solution is expressed in terms of the fundamental solution and Cauchy data, and formulas for generating functions of these solutions. Serious advances were also made in the study of the stability of the Cauchy problem and in the problem of maintaining the Stanley hierarchy of generating functions of several variables. The condition of unimodularity of a rational cone, which is essential in the last two cases, can be rejected in some problems of combinatorial analysis, for example, in the problem of generalized lattice paths in an integer lattice and in the study of vector partition functions with "weight".

The main research tool in the dissertation work of Sreelatha Chandragiri is the method of generating functions, which is a discrete analog of the Fourier transformation. Namely, a solution is sought not for the original Cauchy problem for the difference equation, but for the problem obtained after applying the discrete Fourier transform. From the point of view of enumerative combinatorial analysis, to find the generating function of a sequence describing a certain combinatorial object means to solve the enumeration problems.

The studies presented in the work are of interest not only from the point of view of applying the methods of the theory of generating functions in combinatorial analysis, but also for the general theory of multidimensional difference equations. In particular, not only are the restrictions on the sets where solutions of the difference equation are sought, but also the options for the formulations of the Cauchy problems are expanded.

The dissertation by Sreelatha Chandragiri consists of an introduction and three parts in which the main results are proved. In the introduction, relevance is substantiated, a literature review is given, and the purpose of research is formulated.

Part 1 of the paper is devoted to the Cauchy problem for a multidimensional difference equation with constant coefficients. The main results are Theorem 1.1 and Theorem 1.2, in which formulas are obtained both for the generating function of the solution of the Cauchy problem and for the solution itself.

In Part 2, we define the concept of a vector decomposition function with "weight", prove the identity (Theorem 2.2), relating the generating function for the weight function and the generating function for the vector decomposition functions. As a result, an analogue of the Chaundy-Bullard identity is found (Theorem 2.5)

In part 3, the results of the previous two parts are used to study some classical problems on lattice paths. From Theorem 2.2. an identity for generating functions is obtained (Theorem 3.1), which allows us to find the generating function for Dick paths lying above the main diagonal.

The list of literature used in the dissertation fully reflects the current state of research in the theory of multidimensional difference equations and its application in combinatorial analysis.

The main results are new and represent a significant contribution to the development of this area of mathematical analysis.

Sreelatha Chandragiri mastered the methods necessary for work and successfully applied them to solve the tasks assigned to her. The dissertation is neatly framed, the entire necessary notation is given, and the definitions of the theorem are provided with complete proofs.

In my opinion, the dissertation is of a high quality and S. Chandragiri is to be awarded a PhD degree.

Professor E. Leinartas

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