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September 21, 2015

It was a great pleasure for me to to read the thesis " Continuation of power series by entire and meromorphic interpolation of coefficients" by A. Mkrtchyan. The mathematical content of the thesis is a very nice blend of classical analytic continuation problems "across an arc of the boundary" of the circle of convergence for power series in one complex variable, seen from a new perspective, and their multidimensional analogue (here continuation takes place across much thinner set (poly-arc), of dimension n, of the boundary (whose topological dimension is $2n-1$). In both cases, an immediate consequence is a substantial enlargement of the class of power series(analytic functions) that can be continued "across the part of the boundary". The draw-back here is the fact that the conditions under which the main results are valid are only sufficiency conditions. In addition, the approach developed in the thesis allows the author to have a much clearer picture about the power series that are un-continuable across any point of the boundary.

Let me be more specific. In the first chapter of the thesis the author considers the power series $f(z) = \sum_{n=0}^{\infty} f_n z^n$, whose domain of convergence is the unit disk $D_1 = \{z \in \mathbf{C} : |z| < 1\}$. One is saying that an entire function ϕ interpolates the coefficients of the series if $\phi(n) = f_n$, for all $n \in \mathbf{N}$. In the classical setting it is the growth of its indicator function $h_\phi(\theta)$ (or the growth of the quotient $\frac{h_\phi(\theta)}{\theta}$) which determines the existence and the "size" of the open set so that the analytic extension of f upon it is possible. The originality of the approach presented in the thesis is contained in the fact that Mr.Mkrtchyan replaces entire function interpolating the coefficients of the power series by a specific class of meromorphic functions, also interpolating the coefficients, thus extending the above results to a broader class of interpolating functions. The proposed construction involves meromorphic functions of the form $\psi(\zeta) = \phi(\zeta)\dfrac{\prod\limits_{j=1}^{p} \Gamma(a_j\zeta+b_j)}{\prod\limits_{k=1}^{q} \Gamma(c_k\zeta+d_k)}$ and its associated entire function $\varphi(z) = \phi(\zeta)\dfrac{\prod\limits_{j=1}^{p} a_j^{a_j\zeta}}{\prod\limits_{k=1}^{q} |c_k|^{c_k\zeta}}$. It turns out, according to Theorem 1.1, Theorem 1.2, and Theorem 1.3 proven in the first chapter of the thesis, that powers series extends analytically to larger sets (different in every

theorem) if the indicator function $h_\varphi(\theta)$ and the quantity $l = \sum\limits_{k=1}^{q} |c_k| - \sum\limits_{j=1}^{p} a_j$ satisfy certain growth estimates. The main idea of the proof is to introduce the auxiliary meromorpic function in two complex variables $g(\zeta, z) := \frac{z^\zeta}{e^{2\pi i \zeta}-1}$, which is holomorphic in $z \in \mathbf{C} \setminus \mathbf{R}_+$ and meromorphic in $\zeta \in \mathbf{C}$, and then to compute the limit of integrals $\lim\limits_{m \longrightarrow +\infty} \int\limits_{\partial G_m} \varphi(\zeta) g(\zeta, z) d\zeta$. Here, the contours ∂G_m are suitably chosen rectangles on whose sides the integrand satisfies the appropriate growth conditions involving $\ln |z|$ and $o(\zeta)$. The contours ∂G_m expand to infinity on the right hand-side of the line $\Re\zeta = a$, $\frac{1}{4} < a < \frac{3}{4}$. The key step in the proof is the application of the residue theorem which yields $\int\limits_{\partial G_m} \varphi(\zeta) g(\zeta, z) d\zeta = \sum\limits_{n=1}^{m} \varphi(n) z^n + P(z)$.

Now, taking the limits $m \longrightarrow +\infty$ on the both sides of the above identity, we are led, after some tenuous and delicate computations (variations of Jordan Lemma), to the desired result. This part of the work of Mr.Mkrtchyan requires a deep insight into the mathematical nature of the problem, lot of original and delicate computations and a good command of related bibliography. The presentations of theorems are followed by a number of very instructive examples illustrating the power of the obtained results.

The second chapter of the thesis treats the same type of problems in Several Complex Variables. This part of the thesis is very original, as no results of similar nature have ever appeared in print (at least, as far as, I know). Here one must give a credit to Mr. Mkrtchyan firstly, because he found the "right " multidimensional analogue of one dimensional phenomena in complex analysis, which is not a trivial matter, and secondly for the ingenuity of the arguments he presented in the proofs.

The domain of convergence of power series $\sum\limits_{k \in \mathbf{N}} f_k z^k$ in $\mathbf{C}^n$ is a bounded, logarithmically convex, complete Reinhardt domain. This class of domains contains domains which are not bi-holomorphically equivalent such as poly-disk, ball, hypercone. Thus, from this point of it is natural to consider analytic continuation "across" the poly-arc. It is quite complicated to provide all the intricate technical details of the arguments presented in the second chapter of the thesis under consideration. It is sufficient however to note that Mr. Mcrtchyan uses a good multidimensional analogue for the indicator function due to Ivanov, for global residue theorem (on direct product n-dimensional topological cycles), he uses a global residue theorem for local residues and for the estimates on product of curves the corresponding multidimensional Jordan lemma to be found in the book of Multidimensional Residues by A.Tsikh. The second chapter of thesis is concluded by a series of very instructive examples justifying in the eyes of the reader the importance of the results obtained. Overall, I have the feeling that the second part of the thesis has good potential for further research in the future.

By the results obtained in his thesis, Mr. Mkrtchyan has shown to me that he possesses very good technical skills, that he is able to think originally and deeply and that he is able to combine ideas from different areas of Complex Analysis and Mathematics. The presentation of the results of the thesis is lucid and the details are worked thoroughly, showing to me that Mr. Mrtchyan has matured on the subject.

Overall, the scientific level of the dissertation is up to high international standards and conferring a Ph.D degree to the candidate (provided that the oral part of the defence goes through) seems to be a natural conclusion.

Sincerely yours,

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