# THE CAUCHY PROBLEM FOR DIFFERENCE EQUATIONS IN LATTICE CONES AND GENERATING FUNCTIONS FOR ITS SOLUTIONS

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Difference equations arise in various areas of mathematics, and the theory of linear difference equations in the one-dimensional case is well developed. In particular, it is known that its space of solutions is finite-dimensional. Constant coefficient difference equations in combination with generating functions are a powerful method used in enumerative combinatorial analysis and discrete dynamic systems. For multidimensional difference equations, significant difficulties arise primarily from the fact that their space of solutions is infinite dimensional.

The fundamental papers about the theory of multidimensional difference equations, which proves the existence and uniqueness of their solutions, was written by M. Bousquet-Mélou, M. Petkovšek in 2000. Different formulations of the Cauchy problem for multidimensional difference equations in *unimodular cones* were offered by E. K. Leinartas, A. P. Lyapin, M. S. Apanovich (Rogozina), T. I. Yakovleva (Nekrasova) (2004-2019).

The goal of our research is to consider the Cauchy problem for multidimensional difference equations in *pointed lattice cones*; to give an analogue of the Chaundy-Bullard identity for vector partition functions; to derive generating functions of solutions to restricted lattice path problems by using difference equations with non-constant coefficients.

We use methods of multidimensional complex analysis, including the theory of multidimensional power series and amoebas of algebraic hypersurfaces, and the theory of generating functions (series).

In chapter 1 we consider a variant of the Cauchy problem for a multidimensional difference with constant coefficients, which connected with a lattice path problem in enumerative combinatorial analysis. Such difference equations are used to describe a major class of problems in enumerative combinatorial analysis such as lattice path problems (the Dyck, Motzkin, Schröder and generalized lattice paths).

In chapter 2 we define a vector partition function with weight, which is a generalization of vector partition functions used by M. Brion and M. Vergne (1997), A. V. Pukhlikov and A. G. Khovansky (1993), B. Sturmfels (1995), and derive an identity for generating series of such functions associated with solutions of basic recurrence relation of combinatorial analysis. As a consequence we obtain the generating function of the number of generalized lattice paths and a new version of the Chaundy-Bullard identity for the vector partition function.

In chapter 3 we consider a difference equation in a two-dimensional pointed lattice cone $K$ spanned by a set of vectors including $n$ linearly independent vectors.

**The main results:**

- we obtained formulae in which the generating function of the solution to the Cauchy problem is expressed in terms of generating functions of the Cauchy data and a solution to the Cauchy problem is expressed through its fundamental solution and Cauchy data;
- we obtained a Chaundy-Bullard identity for vector partition functions with weight;
- we obtained the identity for the generating functions (series), based on which we derived generating functions of solutions to restricted lattice path problems.

The thesis comprises the introduction, three chapters, the conclusion, and a list of references, and it has 71 pages.

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